
Network analysis of Zentralblatt MATH data

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Abstract We analyze the data about works (papers, books) from the time period 1990-2010 that are collected in Zentralblatt MATH database. The data were converted into four 2-mode networks (works \times authors, works \times journals, works \times keywords and works \times MSCs) and into a partition of works by publication year. The networks were analyzed using Pajek – a program for analysis and visualization of large networks. We explore the distributions of some properties of works and the collaborations among mathematicians. We also take a closer look at the characteristics of the field of graph theory as were realized with the publications.

Keywords bibliographic networks · two-mode network · large network · collaboration

Mathematics Subject Classification (2000) 01A90 · 00A15 · 91D30 · 68R10 · 93A15

1 Introduction

Bibliographic data allow us to explore the development of an area of research, which authors collaborated most, in which areas of research exist stronger collaboration groups, in which areas authors prefer to work alone or in smaller groups, and much more. Analysis of bibliographic data does not contribute to the areas of research directly, but helps us to understand how they are structured. Network analysis of bibliographic data has been already widely explored, started with E. Garfield (Garfield,

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1979) on. In the paper we intend to present an insight into the field of mathematics as recorded by the Zentralblatt MATH (ZB) database in the decades 1990-2010. The ZB database is maintained by the Berlin editorial office of FIZ Karlsruhe in cooperation with European academies and mathematical institutes.

In cooperation with prof. Bernd Wegner and his associates at FIZ Karlsruhe we obtained in January 2011 the basic data about works (papers, books) for the time period 1990-2010 that are collected in the ZB database. We chose to explore this bibliographic data using network analysis. For computations we used the program Pajek (Batagelj and Mrvar, 2014; De Nooy, Mrvar and Batagelj, 2012), a tool for analysis and visualization of large networks. In this paper we present the results from basic network analyses (statistical information about the data) and identification of important elements (authors, keywords, and journals).

In the paper we first describe the data and discuss some problems encountered in transforming the data into networks. In the third section, different distributions are presented. The analysis of the collaboration network among mathematicians is presented in the fourth section. In the last section we take a closer look on the selected area of mathematics – the graph theory. Analysis of the collaborations among graph theorists, graph theory determining keywords, journals biased toward graph theory and areas of mathematics that overlap with graph theory are presented.

2 Data

The data obtained from the ZB database contain several information about each work. The collection of the information about a single work is called a record and is composed of different fields. Each field has its own 2-character identifier:

- an – identification number of a work (set by ZB),
- ai – unified author's name,
- au – author's name,
- py – publication year,
- cc – classification (Mathematical Subject Classification - MSC) (Wegner and Werner, 2010),
- ti – title,
- ut – keywords,
- is – journal's International Standard Serial Number (ISSN),
- so – journal's title, pages, year,
- se – data about journal (identification number by ZB, whole and short title, ISSN).

An example of a record:

```
an 01714102
ai -; sastre-vazquez.patricia; -
is ISSN 0368-492X
au Us\o-Dom\enech, J.L.; Sastre-Vazquez, P.; Mateu, J.
py 2001
cc *68U20
ti Syntax and first entropic approximation of  $L(M,T)$ . A
  language for ecological modelling.
ut modelling process; text-model based language
```

```
so Kybernetes 30, No.9-10, 1304-1317 (2001).
se 00000540 Kybernetes Kybernetes 0368-492X
```

2.1 Problems with the data

Data about works are entered in the ZB database by editors. The most common problem are the missing data. Some papers and books do not have all types of information entered. If the missing information is needed in any of analyses this leads to additional problems.

The non ASCII characters in the text are represented by T_EX commands. The problem is the nonuniform use of T_EX. For example

```
au Must\u{a}\c{t}a, Costic\u{a}
au Must\u a\c ta, Costic\u a
```

are two different writings of the name of the same author. To solve the problem we need to write a script that recognizes all the different writings of the same character.

Author's names are only partially unified in the ZB database. Some authors have unified names, but others do not, or even have several of them (synonymy). This problem is not easily solvable, because someone would need to look at the list of all authors and their unified names and make the necessary corrections. There exist authors with their names written in more than one variant. For example Mankoč Borštnik, Norma Susana is written as

```
Bor\v stnik, N. S. Manko\v c
Manko\v c Bor\v stnik, N.
Manko\v c-Bor\v stnik, Norma
Manko\v c Bor\v stnik, Norma Susana
Mankoc-Borstnik, N.S.
Manko\v c Bor\v stnik, N.S.
```

The unification is also a problem for another reason. Some authors have very similar or even the same names (homonymy). They might also have the same unification of their name in ZB database, which results in the problem of distinguishing between these authors.

Not only the authors' names are the problem, but also the keywords. Because not all of the works have assigned keywords, we extracted also words from the title and considered them as keywords. (Real) keywords are actually phrases consisting of at least one word. We splitted phrases into words and removed the stop words. Related keywords were unified using lemmatization (MontyLingua package in Python). For example, keywords algebra and algebras were unified.

Journals in ZB have identification numbers. This, in principle, solves the unique identification problem. But we did find one journal with two identifiers during analyses:

```
se 00000552 Match Match 0340-6253
se 00003047 MATCH - Communications in Mathematical and in
    Computer Chemistry MATCH Commun. Math. Comput.
    Chem. 0340-6253
```

We treat them as a single journal.

Journals are changing through time – a new journal is ‘born’, a journal ‘dies’, some journals are merged into journal, a journal is split into some journals, a journal changes the title, etc. Some journals had just changed the title. Because of such changes they appear as different journals in the database. We merged different appearances of a journal that changed the title into one journal.

2.2 Preparation of the data

With a special program written in Python we converted the data into the Pajek format (Batagelj and Mrvar, 2014). We obtained four compatible 2-mode networks and a partition of works by their publication year.

A network is a structure $\mathcal{N} = (\mathcal{V}, \mathcal{L}, w)$, which consists of a set of nodes \mathcal{V} , a set of links among nodes \mathcal{L} and a weight function $w : \mathcal{L} \rightarrow \mathbb{R}$, that determines the weights of the links. A network is called a 2-mode network, if the set of nodes \mathcal{V} is partitioned into two disjoint subsets and each link has its end nodes in different subsets.

In our data, the first subset of nodes in all four networks consists of identifiers of **works** and is denoted by \mathcal{W} . Nodes in the second subset represent one of the following:

- \mathcal{A} – a set of **authors**,
- \mathcal{J} – a set of **journals**,
- \mathcal{K} – a set of **keywords**,
- \mathcal{M} – a set of **MSCs** (mathematical subject classifications).

Information was extracted from the records of all works. The identifiers of works were extracted from the field `an`, journals from the field `se` and MSCs from the field `cc`. Keywords were extracted from the fields `ut` and `ti` as phrases and then decomposed into words and unified using lemmatization. Names of authors were extracted from the field `ai`. If the author’s ZB-unified name does not exist, his/her name was extracted from the field `au` and unified into ZB-names-like form.

We expect that most of the important mathematicians have their unified name. For the rest, we decided to treat the synonymy/homonymy as a kind of noise and reconsider them in cases when they appear as ‘duplicates’ in the results.

Links in all produced networks are directed – **arcs**, and they link each work to some representatives in the second set. The co-authorship network of works \times authors $\mathbf{WA} = ((\mathcal{W}, \mathcal{A}), \mathcal{L}, w)$ is a network in which each work is linked to all of its authors, $(p, i) \in \mathcal{L} \Leftrightarrow i$ is an author of work p . The other three networks are defined in a similar way – works are linked to journals, keywords and MSCs in networks works \times journals \mathbf{WJ} , works \times keywords \mathbf{WK} , and works \times classifications \mathbf{WM} , respectively. We will also use a simplified notation for a transposed network: the transposed network of the network \mathbf{WA} is denoted with $\mathbf{AW} \equiv \mathbf{WA}^T$ and is obtained from \mathbf{WA} by changing the directions of all its arcs. The sizes of all four networks are listed in Table 1.

As mentioned before, we had problems with the notion of an author. Some of them appeared twice or even more times under different names in the network \mathbf{WA}

Table 1 Sizes of 2-mode networks.

Network	WA	WJ	WK	WM
Size of the first set	1,339,201	1,339,201	1,339,201	1,339,201
Size of the second set	557,104	3,158	143,513	12,390
Number of arcs	2,550,437	1,331,036	15,062,377	3,370,820

because of only the partial unification of their names. We made a partition of the set of authors by collecting different appearances of the same author. For example O'Regan, Donal is once written as `oregan.donal` and another time as `o'regan.d`. This author has a ZB-unified name `oregan.donal`, but sometimes his unified name is not written and in such cases our program for the data conversion creates it from the full author's name – O'Regan, Donal get unified-like name `o'regan.d`. Another author with similar problem is Pečarić, Josip E. His unified ZB-name is `pecaric.josip-e`, sometimes unified name is not written and we get `pecaric.j` and `pecaric.j-e` because of two different writings of his full name: Pečarić, J. and Pečarić, J. E. Yet another source of problems is the writing of Eastern European surnames: Krachkovskij, A. P., and Krachkovskii, A. P., are probably representing the same author.

The partition of author's names solved the unification problem only partially. We also used the AMS identification of authors (TePaske-King and Richert, 2001) for help with the unification problem. All the following analyses were made after the additional unification of different appearances of the same author names.

We also solved the problem with journals. Different names of the same journal were replaced by a single name – from 3158 journal names we obtained 2665 unique journal names.

3 Distributions of properties of works

We examined degrees of nodes in the obtained networks to determine distributions of different data. With outdegrees of nodes in the set of works in networks **WA**, **WK**, **WM** we obtained the distributions of works by number of authors, keywords and classifications. Each work is supposed to be published in at most one journal. None of the works in the database was published in more than one journal. There are 8165 works that have no journal determined.

The distribution of works published in the time period 1990-2010 that are indexed in Zentralblatt MATH by their publication year is shown in Fig. 1. We see that the number of indexed works is growing – in 20 years it has almost doubled. The decrease in the years 2009 and 2010 is due to works that are still to be indexed.

For a work $p \in \mathcal{W}$ its $\text{outdeg}(p)$ in the network **WA** is a number of authors of the work p . Distribution of works by a number of authors, $F(d) = |\{p \in \mathcal{W} : \text{outdeg}(p) = d\}|$ = number of works each having exactly d authors, is shown in Fig. 2 in the top diagram. The curves in all diagrams in Fig. 2 are gaussian kernel density estimates of the distributions. More than $\frac{1}{3}$ of all works (37.99%) was written by a single author and another $\frac{1}{3}$ of all works (34.60%) by a pair of authors. 2383 works do not have any

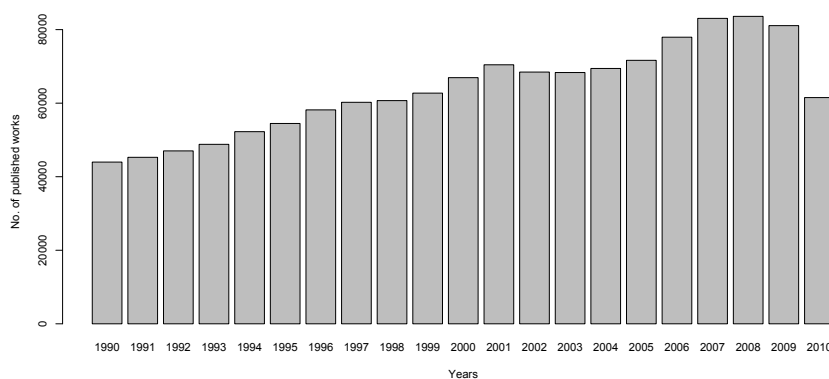


Fig. 1 Distribution of works by their publication year

author attributed and on the other hand some works have large number of co-authors – even 70 co-authors per work. Works with the largest number of co-authors are:

- 70 co-authors – Aderholz, M. et al.: Distributed applications monitoring at system and network level. *Comput. Phys. Commun.* 140, No.1-2, 219-225 (2001).
- 38 co-authors – Bridle, S. et al.: Handbook for the GREAT08 challenge: an image analysis competition for cosmological lensing. *Ann. Appl. Stat.* 3, No. 1, 6-37 (2009).
- 35 co-authors – Regan, S.P. et al.: Direct-drive inertial confinement fusion implosions on omega. *Astrophys. Space Sci.* 298, No. 1-2, 227-233 (2005).

The distribution of works by a number of keywords is shown in the center of Fig. 2. Note that our keywords were produced from the keywords and the title, as explained earlier. This distribution is quite flat. Approximately 50% of all works have the number of keywords between 10 and 18. Works with the largest number of keywords are:

- 71 keywords – Baianu, I.C. et al.: Algebraic topology foundations of supersymmetry and symmetry breaking in quantum field theory and quantum gravity: a review. *SIGMA, Symmetry Integrability Geom. Methods Appl.* 5, Paper 051, 70 p., electronic only (2009).
- 69 keywords – Dutta, H.: On some sequence spaces generated by $\Delta(r)$ – and Δr – difference of infinite matrices. *Int. J. Open Probl. Comput. Sci. Math., IJOPCM* 2, No. 4, 496-504 (2009).
- 68 keywords – Cheng B. and Tong, H.: On consistent nonparametric order determination and chaos. *J. R. Stat. Soc., Ser. B* 54, No.2, 427-449 (1992).

Distribution of works by number of MSCs is shown at the bottom of Fig. 2. Approximately one third of all works (30.92%) were classified with two MSCs and approximately 40% of all works (43.57%) were classified with one or three MSCs. Works with largest numbers of MSCs are:

- 21 MSCs – Auroux, D. et al: Report 35/2006: Four-dimensional Manifolds (August 6th – August 12th, 2006). *Oberwolfach Rep.* 3, No. 3, 2059-2140 (2006).

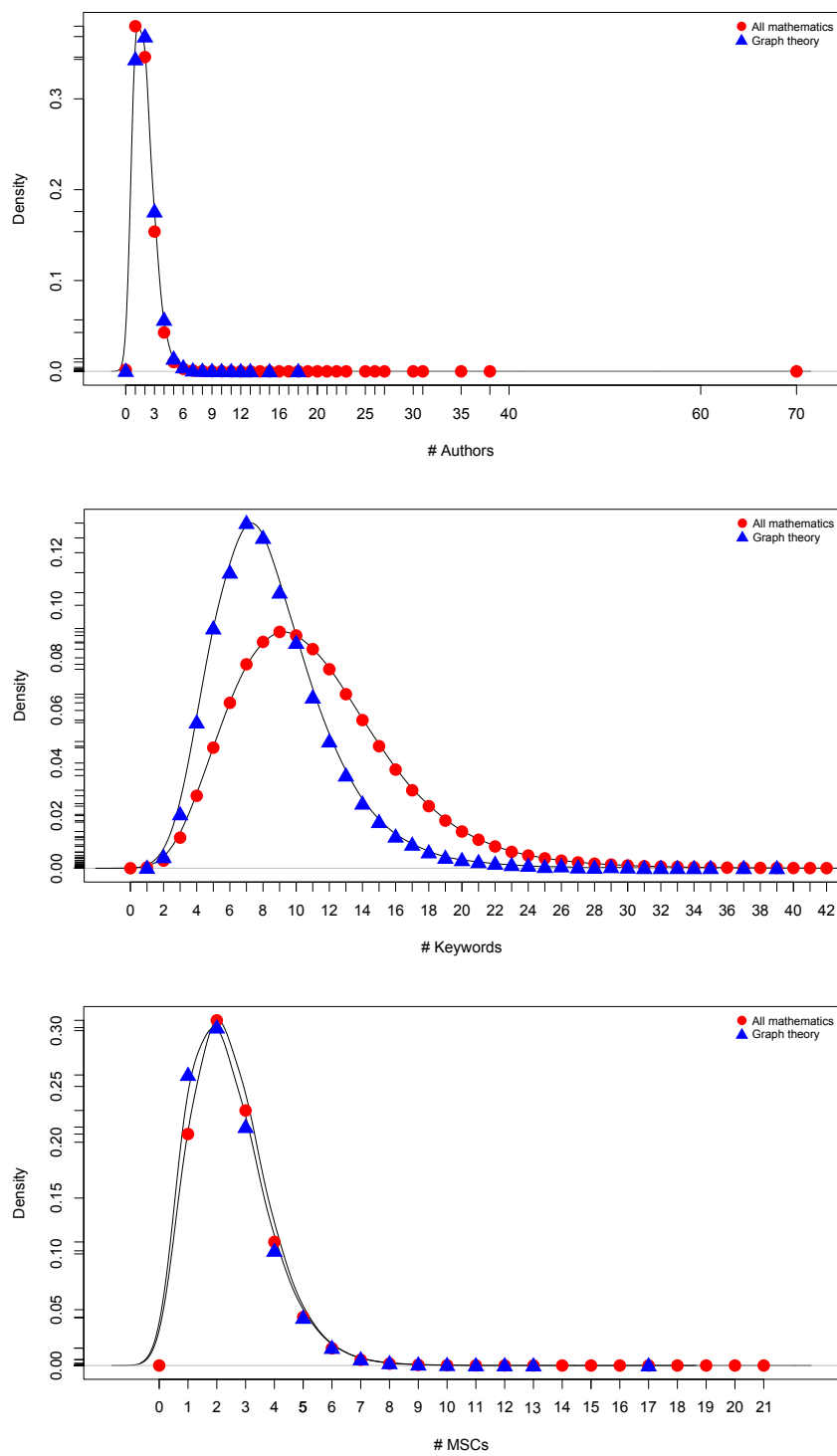


Fig. 2 Distributions of works by the number of authors, keywords and MSC classifications. These figures include also the same distributions for only the field of graph theory which is the topic of Section 5.

- 21 MSCs – Dechevsky, L.T.: Concluding remarks to paper “properties of function spaces generated by the averaged moduli of smoothness”. Int. J. Pure Appl. Math. 49, No. 1, 147-152 (2008).
- 20 MSCs – Aubin, J.-P.: A survey of viability theory. SIAM J. Control Optimization 28, No.4, 749-788 (1990).

It turns out that all distributions in Fig. 2 can be very well approximated by the lognormal distribution, gamma distribution and also by the generalized reciprocal power exponential curve $c * (x+d)^{\frac{a}{b+x}}$. In all cases we get the best fit with gamma distribution. The results are given in Table 2. For technical details see Subsection 2.5.2 *Fitting distributions* in Batagelj, Doreian, Ferligoj and Kejžar (2014).

Table 2 Fitting the gamma distribution $c \cdot \Gamma(x, a, b)$.

Distribution	c	a	b	residual SS
Authors (all)	$1.342 \cdot 10^6$	3.621	1.905	99893627
Authors (graph theory)	$4.056 \cdot 10^4$	3.876	1.920	23743
Keywords (all)	$1.333 \cdot 10^6$	5.561	0.498	10079394
Keywords (graph theory)	$4.006 \cdot 10^4$	6.960	0.832	188488
MSCs (all)	$1.345 \cdot 10^6$	3.718	1.457	273424172
MSCs (graph theory)	$4.136 \cdot 10^4$	3.078	1.261	357960

In addition to examining the distributions of degrees of nodes in the first subset of the two-mode networks, we examined the distributions of degrees of nodes in the second subset as well. The distribution of authors by number of works they co-authored is shown in Fig. 3 in the top figure. For example a dot in the upper left corner represents 271013 authors that each co-authored only one work in the time-period of 1990-2010. Dots in the lower right corner are representing Ballico, Edoardo with 967 works co-authored in a given time-period, O'Regan, Donal with 821 works, Pečarić, Josip with 606 works, Agarwal, Ravi P. with 598 works and Srivastava, H.M. with 582 works co-authored in a given time-period as indexed in the ZB database. One can notice that Lotka's law holds for authors of up to 16 works.

The distribution of keywords by the number of works they describe is shown in Fig. 3 in the second figure. The dot in the upper left corner represents 72314 keywords that were used in the description of works only once in the time-period 1990-2010. Dots in lower right corner represent most commonly used keywords: equation (188483 times used), problem (152514), function (129957), method (128740), model (123448), space (112000), solution (109068), linear (76241), theory (75873) and finite (75398). These words are actually the most common words in mathematics. The shape of the distribution of keywords by the number of works in the second figure in Fig. 3 is typical for empirical distributions of quantities following the power law $f_n = cn^{-\alpha}$. Using the function `power.law.fit` in the R package `igraph` that implements M. Newman's procedure described in Clauset, Shalizi and Newman (2009) we get $\alpha = 1.85$. To visually check the power law nature of the distribution we can use the property that, for $\alpha > 1$, if the sequence (f_n) obeys the power law then it is also obeyed by the sequence (g_n) defined as $g_n = \sum_{i=n}^{\infty} f_i \simeq Cn^{1-\alpha}$ as

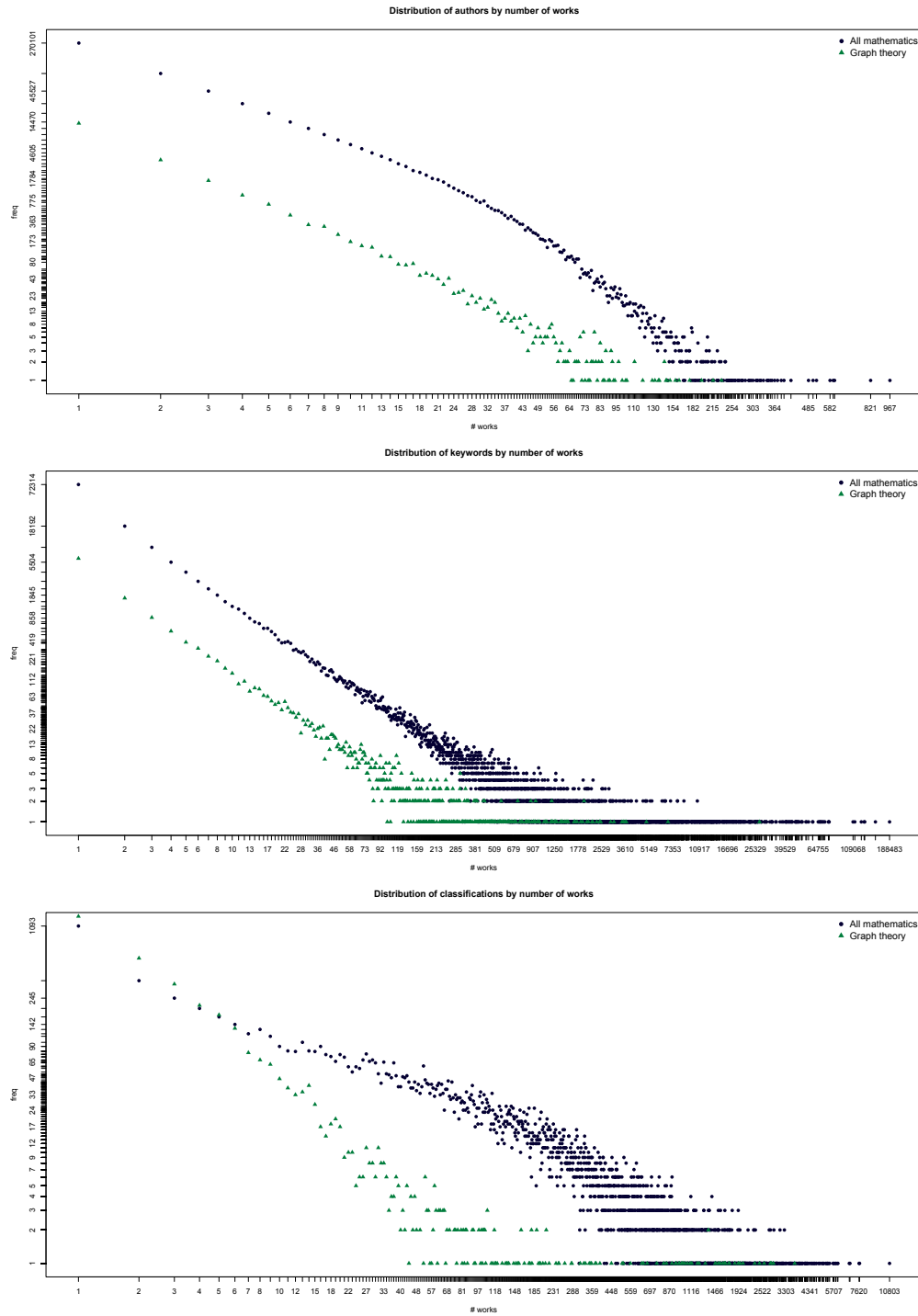


Fig. 3 From top to bottom: Frequency distribution of authors by the number of co-authored works in a double logarithmic scale; Frequency distribution of keywords by the number of works using a keyword in their description in a double logarithmic scale; Frequency distribution of MSCs by the number of classified works in a double logarithmic scale. These figures also include the same distributions for only the field of graph theory which is the topic of Section 5

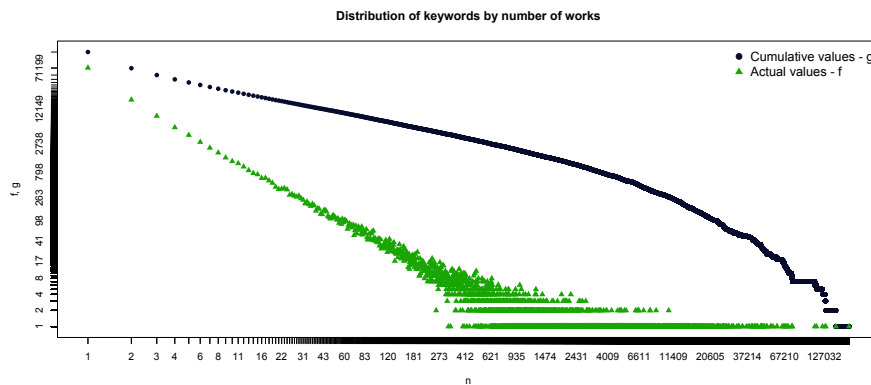


Fig. 4 Sequences f and g for the distribution of keywords by the number of works.

is presented with Eq. 4.38 in Barabási, A-L.: *Network Science*, 2014, available at <http://barabasilab.com/networksciencebook>. Therefore in the joint picture of both sequences in double logarithmic scale we should get two ‘lines’. For the distribution of keywords by the number of works this is not the case as can be seen on Figure 4. The distribution doesn’t obey the power law.

The distribution of MSCs by the number of works that were classified with a given MSC is displayed in Fig. 3 in the bottom figure. Every work is classified with one primary and maybe some secondary MSCs. The same primary and secondary MSC (for example 74S05 and \star 74S05) are represented with one dot. Each dot represents on the y-axis determined number of MSCs that were used for classification of on x-axis determined number of works. The dot in the upper left corner represents 1093 MSCs, that were included in the classification of only one work in the time-period of 1990-2010. MSCs in Table 3 are the most frequently used MSCs. These MSCs are represented with dots in the lower right corner of Fig. 3. The most frequently used primary MSCs are listed in Table 4.

Table 3 Table of the most popular MSCs.

MSC code	2-char MSC name	MSC name	No. of works
80A20	Classical thermodynamics, heat transfer	Heat and mass transfer, heat flow	13279
74S05	Mechanics of deformable solids	Finite element methods	13271
68T05	Computer science	Learning and adaptive systems	11775
35B40	Biology and other natural sciences	Molecular structure	9338
62P10	Operations research, mathematical programming	Combinatorial optimization	8935
35Q53	Partial differential equations	KdV-like equations	8528
91B28	Game theory, economics, social and behavioral sciences	Finance, portfolios, investment	8366
76D05	Fluid mechanics	Navier-Stokes equations	8207
65N30	Numerical analysis	Finite elements, Rayleigh-Ritz and Galerkin methods, finite methods	7976
62M10	Statistics	Time series, auto-correlation, regression, etc.	7926

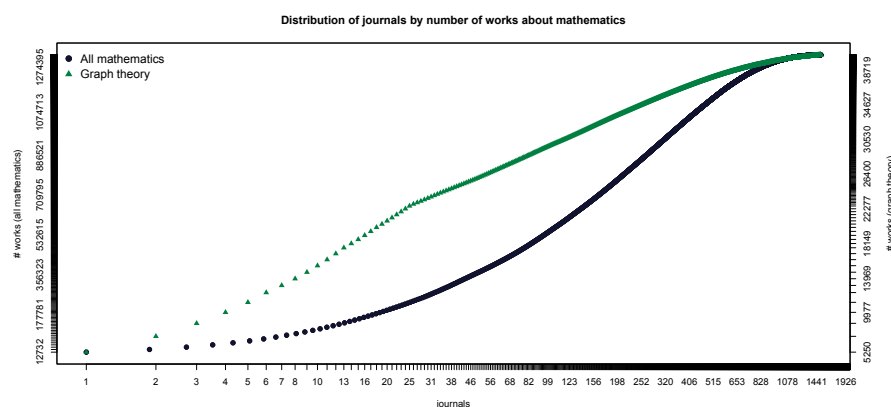
Table 4 Table of the most popular primary MSCs.

MSC code	2-char MSC name	MSC name	No. of works
74S05	Mechanics of deformable solids	Finite element methods	7620
01A70	History and biography	Biographies, obituaries, personalia, bibliographies	5983
68T05	Computer science	Learning and adaptive systems	5943
90B35	Operations research, mathematical programming	Scheduling theory, deterministic	5707
91B28	Game theory, economics, social and behavioral sciences	Finance, portfolios, investment	5386
62P10	Operations research, mathematical programming	Combinatorial optimization	5316
68U99	Computer science	None of the above, but in this section	5193
62-99	Statistics	Other applications	5073
35Q53	Partial differential equations	KdV-like equations	4792
90B30	Operations research, mathematical programming	Production models	4656

The sequence of journals from the time-period of 1990-2010 is shown in Fig. 5 in a shape of Bradford's graph. Values on the x-axis are shown in a logarithmic scale. Each dot represents one journal. Values on the y-axis are cumulative sums of indexed works in journals. Journals on the left contain the largest number of indexed works. These journals are:

- 32132 works – Journal of Physics A: Mathematical and General
- 25464 works – Journal of Mathematical Analysis and Applications
- 20564 works – Proceedings of the American Mathematical Society
- 20322 works – Applied Mathematics and Computation
- 18110 works – European Journal of Operational Research

Journals in the right corner have published just two works indexed in ZB in a given time-period. Some of these journals are: Journal of the History of Economic Thought, The Montana Mathematics Enthusiast, Journal of Mathematics Education, Vestnik Moskovskogo Universiteta. Seriya VI, International Journal of Energy, Environment and Economics, etc.

**Fig. 5** Journals in Bradford's graph form. This figure also includes the same distribution for the field of graph theory only which is the topic of Section 5.

4 Collaboration Network

The collaboration among mathematicians can be explored through the collaboration network. The set of nodes in the collaboration network is the set of authors and two authors are linked if they co-authored at least one work.

We determined the collaboration network as presented in Batagelj and Cerinšek (2013): $\mathbf{Co} = \mathbf{AW} * \mathbf{WA}$. The value of a link between two authors is equal to the number of works they have in common. The 20 authors with the highest numbers of co-authors are presented in Table 5. Since a name can belong to different authors, we checked the authors' names in the MathSciNet Authors Search (TePaske-King and Richert, 2001). The names in Table 5 are divided into two columns – the names in the first column represent a single author and the names in the second column can represent more authors. The third number is the number of known mathematicians with this name.

Table 5 The authors with largest number of co-authors.

i	Author	No. of co-authors	Author	No. of co-authors	No. of authors with this name in AMS
1	Srivastava, Hari Mohan	347	Wang, Wei	463	282
2	Chen, Guanrong	341	et al.	316	
3	Alon, Noga	288	Zhang, Wei	293	228
4	Pardalos, Panos M.	212	Li, Wei	277	193
5	Il'in, V.A.	195	Li, Jun	244	157
6			Wang, Hui	232	132
7			Wang, Yong	224	164
8			Wang, Jun	223	166
9			Zhang, Li	218	465
10			Li, Li	217	324
11			Wang, J.	208	1144
12			Li, Gang	199	42
13			Zhang, Jun	199	130
14			Li, Ming	193	133
15			Wang, Y.	192	1377

The subset of the most collaborative authors can be determined with p_S -cores (Batagelj and Zaveršnik, 2011). In a network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, w)$ the subset $\mathcal{U} \subseteq \mathcal{V}$ is a p_S -core at level $t \in \mathbb{R}^+$ iff

- for each $v \in \mathcal{U} : p_s(v, \mathcal{U}) = \sum_{u \in N(v) \cap \mathcal{U}} w(v, u) \geq t$,
- \mathcal{U} is maximal.

A p_S -core at level t in a collaboration network is such a subnetwork in which each author's contribution to joint works with some other authors in this subnetwork is at least t . A lot of published works does not necessarily mean a larger collaborativeness for their author. In a computation of p_S -cores we are summing up the values of links. To neutralize the over-representation of works with many co-authors in the resulting collaboration network we used the normalized co-authorship network \mathbf{N} in the computation of a collaboration network (Batagelj and Cerinšek, 2013): $\mathbf{N} = \text{diag} \left(\frac{1}{\max(1, \deg(p))} \right) \cdot \mathbf{WA}$. In a network \mathbf{N} the values of links from a work to all of its co-authors are equal and they sum up to 1. In Batagelj and Cerinšek (2013) we

calculated the normalized network $\mathbf{Ct} = \mathbf{N}^T * \mathbf{N}$ to get the contributions of authors to their works. Each work with k authors adds to the network \mathbf{Ct} a corresponding complete directed graph (with loops) on k nodes. Each of its arcs has the weight $\frac{1}{k^2}$. For the analysis of the ZB data we used a slightly changed normalized collaboration network \mathbf{Ct}' which is an undirected network without loops obtained as the sum of complete undirected graphs. Each edge of a complete graph for a work with k authors has the weight $\frac{2}{k \cdot (k-1)}$. The network \mathbf{Ct}' can be obtained as a symmetrization of $\mathbf{N}^T * \mathbf{N}'$ and setting the diagonal values to 0, where $\mathbf{N}' = \text{diag} \left(\frac{1}{\max(1, \deg(w)-1)} \right) \cdot \mathbf{W}\mathbf{A}$. With this we neutralize works with many co-authors.

Fig. 6 shows a p_5 -core at level $t = 30$ in a normalized collaboration network from the ZB data. In the lower half we see mostly pairs of authors that represent authors that collaborate in ‘tandems’. Another interesting thing in this p_5 -core is the large group of authors on the left. In this group one can notice stronger links between some authors – darker and thicker links represent larger contribution to the works in common. Ten strongest collaboration pairs in this p_5 -core are listed in Table 6.

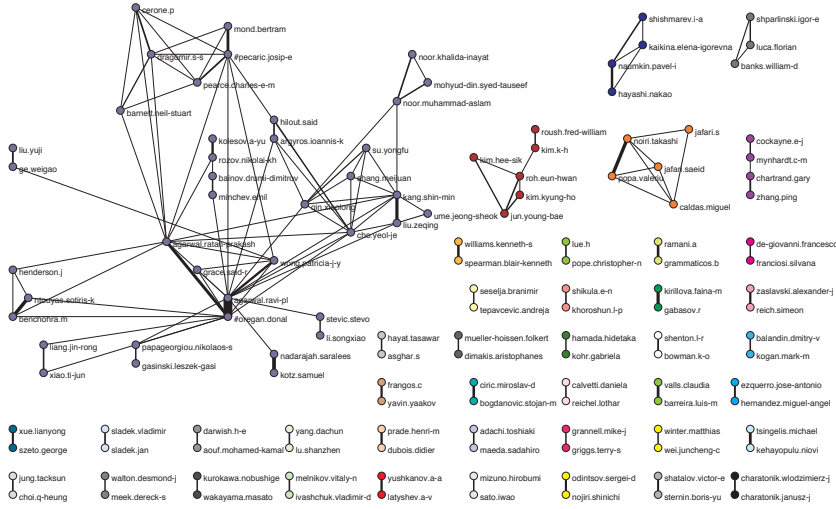


Fig. 6 The p_5 -core at level $t = 30$ in the collaboration network \mathbf{Ct}'

The productivity of an author can be defined in different ways. Let us say, that the author is more self-sufficient if he/she has the largest value of the self-contribution to the works he/she co-authored. This information can be obtained from network $\mathbf{Cn} = \mathbf{A}\mathbf{W} * \mathbf{N}$ (Batagelj and Cerinšek, 2013). The link value cn_{ij} in this collaboration network is equal to the contribution of the author i to the works he/she wrote together with the author j . The weight of a loop cn_{ii} is equal to the self-contribution of the author i to all works that he/she co-authored and is equal to the fractional productivity defined in (Price and Beaver, 1966).

Table 6 List of the strongest collaboration pairs with the values of links between them in the third column.

i	First author	Second author	Link value
1	Agarwal, Ravi P.	O'Regan, Donal	49.46
2	Kotz, Samuel	Nadarajah, Saralees	20.97
3	Ntouyas, Sotiris K.	Benchohra, Mouffak	19.69
4	Popa, Valeriu	Noiri, Takashi	19.00
5	Gabasov, Rafail	Kirillova, Faina Mihaïlovna	17.88
6	Liu, Zeqing	Kang, Shin Min	17.72
7	O'Regan, Donal	Agarwal, Ratan Prakash	16.85
8	Pečarić, Josip E.	Mond, Bertram	15.57
9	Kehayopulu, Niovi	Tsingelis, Michael	15.53
10	Barreira, Luis M.	Valls, Claudia	15.28

We define the self-sufficiency index S_i as the proportion of author's self-contribution and the total number of his/her works. The *collaborativeness* index K_i is defined as complementary value to the self-sufficiency index, $K_i = 1 - S_i$ (Batagelj and Cerinšek, 2013), that is closely related to the collaborative coefficient (Ajiferuke, Burrell and Tague, 1988).

The 'best' mathematicians (the most productive) are listed in Table 7 with their self-contributions denoted as cn_{ii} in the second column. The total number of his/her published works is listed in the third column and the collaborativeness index is listed in the fourth column. Only three names in this list can represent more than one author (as checked in the AMS Authors Search): Evans, D. J., Wang, Wei, and Zhou, Yong.

The mathematicians with the largest number of works are not necessarily on the top of the list of the "best" mathematicians. The 'best' mathematicians have a lot of works written and also a large contribution to those works. If the self-contribution of an author is almost equal to his/her total number of works, he/she tends to work alone or in small groups. The first two authors with the largest number of works are also the 'best' authors – Edoardo Ballico and Donal O'Regan. But there is a large difference between them – Edoardo Ballico tends to work alone and Donald O'Regan tends to work in groups. Ballico's self-contribution value is almost equal to his total number of works (90%) and O'Regan's self-contribution value is equal to a half of his total number of works (53%). Next in the line by the total number of works are Josip E. Pečarić, Mohan Hari Srivastava, and Weigao Ge. The most collaborative among the authors in Table 7 are Guanrong Chen, Ravi P. Agarwal, Lansun Chen, Jaume Llibre, and Josip E. Pečarić.

5 05Cxx Graph theory

Graph theory is a subdiscipline of combinatorics 05 and its three-char MSC is 05C. We can look at graph theory as pure or with its applications – MSCs from other mathematical disciplines which are by content connected to graph theory can be included.

For further analysis we took the network \mathbf{WM}_3 which is a shrunken version of the network \mathbf{WM} : the set of 5-char MSCs is shrunken into a set of 3-char MSCs. A

Table 7 List of the ‘best’ mathematicians in 1990-2010.

i	Author	cn_{ij}	Total	K_i
1	Ballico, Edoardo	865.58	967	0.105
2	O'Regan, Donal	432.80	821	0.470
3	Argyros, Ioannis Konstantinos	353.17	373	0.053
4	Shelah, Saharon	302.49	519	0.417
5	Verma, Ram U.	297.58	314	0.052
6	Srivastava, Hari Mohan	267.17	582	0.541
7	Pečarić, Josip E.	265.28	608	0.564
8	Papageorgiou, Nikolaos S.	263.92	418	0.369
9	Pachpatte, Baburao G.	255.00	261	0.023
10	Maslov, Victor P.	247.08	324	0.237
11	Agarwal, Ravi P.	244.12	598	0.592
12	Wazwaz, Abdul-Majid	242.67	254	0.045
13	Noor, Muhammad Aslam	241.00	351	0.313
14	Jun, Young Bae	239.40	480	0.501
15	Dragomir, S. S.	232.73	364	0.361
16	Le, Maohua	226.17	242	0.065
17	Ge, Weigao	225.78	504	0.552
18	Nadarajah, Saralees	213.70	315	0.322
19	Ramm, Alexander G.	211.92	270	0.215
20	Stević, Stevo	206.20	257	0.198
21	Gamkrelidze, R.V.	190.65	268	0.289
22	Zaslavski, Alexander J.	187.50	236	0.206
23	El Naschie, Mohamed Saladin	183.08	186	0.016
24	Evans, D. J.	182.88	356	0.486
25	Wang, Wei	178.79	394	0.546
26	Nazarov, Serguei A.	176.95	266	0.335
27	Chen, Huanyin	175.50	206	0.148
28	Alzer, Horst	171.92	198	0.132
29	Luca, Florian	167.62	292	0.426
30	Danchev, Peter Vassilev	166.00	170	0.024
31	Guo, Boling	165.08	341	0.516
32	Nishimoto, Katsuyuki	163.42	213	0.233
33	Chajda, Ivan	162.00	244	0.336
34	Shparlinski, Igor E.	161.50	292	0.447
35	Owa, Shigeyoshi	161.06	331	0.513
36	Anastassiou, George A.	157.08	200	0.215
37	Noiri, Takashi	152.25	321	0.526
38	Ikramov, Kh.D.	151.08	198	0.237
39	Jakubík, Ján	150.83	158	0.045
40	Zhou, Yong	150.03	245	0.388
41	Chen, Guanrong	149.54	385	0.612
42	Biswas, Indranil	148.83	224	0.336
43	Llibre, Jaume	148.77	345	0.569
44	Khrennikov, Andrei Yu.	148.52	192	0.226
45	Hall, Peter G.	145.45	294	0.505
46	Chen, Lansun	141.03	344	0.590
47	Aouf, Mohamed Kamal	139.99	251	0.442
48	Chen, Bang-Yen	138.00	177	0.220
49	Park, Sehie	137.25	166	0.173
50	Alon, Noga	136.37	298	0.542

combination of this network with other networks allows us to analyze the field of graph theory as it can be seen through published works.

To see which journals published the largest amount of indexed works about graph theory, we need a network **WJ** and a network **WM₃**. The values of links in the second network might be larger than one – a work can have more MSCs with the same first 3 chars determined. We changed these values of links to 1 and multiplied networks **WJ** and **WM₃** to get the network **JM₃** = **JW** * $b(\mathbf{WM}_3)$, where $b(\mathcal{N})$ is the binarized version of network \mathcal{N} . The link value $jm3_{jc}$ in this network is equal to the number of indexed works that were published in a journal j and were classified with a classification c . We normalized this network in a similar way as we normalized the **WA** network to get the normalized collaboration network: $n(\mathbf{JM}_3) = \text{diag}\left(\frac{1}{\text{weighted deg}(j)}\right) \mathbf{JM}_3$. The weighted degree of a node is equal to the sum of incident links values. The sum of incident links' values of each journal in the network $n(\mathbf{JM}_3)$ is now equal to 1.

We took a look at link values from journals to the graph theory classifications. The link values represent the percentages of indexed works in the ZB published in the selected journal that are classified with a graph theory MSC. In the left column of the Table 8 are listed the journals with the largest percentages of such works. In the right column of the Table 8 are listed the journals that have largest percentages of indexed works in the ZB about graph theory with its applications included. MSCs that represent graph theory's applications are 68R10, 81Q30, 81T15, 82B20, 82C20, 90C35, 92E10, 94C15, 05E30, 57M15, 57M25, 20F65, 90B10, 05B30, 05D10, 91A43, 91A46, 60B20, 91D30, 68R10, 68W05, 81Q30, 81T15, 82B20, 82C20, 90C35, 92E10, 94C15, and all that start with 90B.

The difference in both lists is easily seen. There is one journal (The European Physical Journal B. Condensed Matter) from which only works with at least one classification from graph theory or its applications were included in the ZB.

Another way of determining journals that published a lot of works about graph theory is using biases (Grcar, 2010). The bias of a journal for or against any branch of mathematics is

$$\text{bias} = \log_2 \frac{\text{fraction of works about the subject in the journal}}{\text{fraction of works about the subject in all of mathematics}}.$$

This value basically tells us if some journal is favoring a selected branch or subject of mathematics (positive value) or if it is hindering it (negative value). If the value of bias is equal to zero, the journal published relatively as many works about the selected branch or subject of mathematics as all journals together did.

In Table 9 the journals with the largest positive biases for the graph theory are listed, and in Table 10 are the journals with the largest negative biases for the graph theory. We include in the calculation of the bias value only the journals that published at least 50 works indexed in the ZB database. An author can use the bias value for his/her topic to determine the best journals for submitting his/her work. A positive bias of a journal for the selected topic means that this journal is more likely to publish a work about this topic; and a negative bias of a journal for a topic means that this journal is more likely to reject a work about this topic.

Table 8 Journals with the largest percentages of indexed works about graph theory in the time-period 1990-2010: pure graph theory (left), graph theory with applications (right).

Pure graph theory		Graph theory and its applications	
Journal of Graph Theory (0364-9024, 1097-0118)	89.15%	The European Physical Journal B. Condensed Matter (1434-6028)	100.00%
AKCE International Journal of Graphs and Combinatorics (0972-8600)	82.55%	Journal of Graph Theory (0364-9024, 1097-0118)	95.87%
Journal of Combinatorial Theory. Series B (0095-8956)	68.85%	AKCE International Journal of Graphs and Combinatorics (0972-8600)	88.89%
Graphs and Combinatorics (0911-0119, 1435-5914)	59.45%	Journal of Combinatorial Theory. Series B (0095-8956)	83.20%
Ars Combinatoria (0381-7032)	50.04%	ITS Journal (1024-8072)	80.77%
The Australasian Journal of Combinatorics (1034-4942)	46.96%	International Journal of Flexible Manufacturing Systems (0920-6299, 1572-9370)	79.44%
JCMCC. The Journal of Combinatorial Mathematics and Combinatorial Computing (0835-3026)	43.81%	Graphs and Combinatorics (0911-0119, 1435-5914)	76.49%
Ars Mathematica Contemporanea (1855-3966, 1855-3974)	42.86%	International Journal of Production Research (0020-7543, 1366-588X)	76.48%
Congressus Numerantium (0384-9864)	42.81%	Match (0340-6253)	75.05%
Match (0340-6253)	42.80%	Journal of Graph Algorithms and Applications (1526-1719)	74.72%
Discrete Mathematics (0012-365X)	42.39%	Location Science (0966-8349)	74.29%
Bulletin of the Institute of Combinatorics and its Applications (1183-1278)	42.28%	Journal of Scheduling (1094-6136, 1099-1425)	68.84%
Advances and Applications in Discrete Mathematics (0974-1658)	42.27%	Journal of Interconnection Networks (0219-2659)	66.67%
Combinatorica (0209-9683)	33.99%	Studies in Locational Analysis (1105-5162)	66.48%
Combinatorics, Probability and Computing (0963-5483, 1469-2163)	33.76%	Networks (0028-3045, 1097-0037)	66.04%
College Mathematics Journal (0746-8342)	33.33%	Transportation Science (0041-1655)	63.77%
International Journal of Mathematical Combinatorics (1937-1055)	29.29%	Networks and Spatial Economics (1566-113X, 1572-9427)	62.00%
Random Structures & Algorithms (1042-9832, 1098-2418)	29.06%	The Australasian Journal of Combinatorics (1034-4942)	60.31%
Discussiones Mathematicae. Graph Theory (1234-3099)	28.95%	Ars Combinatoria (0381-7032)	59.96%
Journal of Combinatorics, Information & System Sciences (0250-9628)	28.24%	JCMCC. The Journal of Combinatorial Mathematics and Combinatorial Computing (0835-3026)	59.87%

Table 9 Journals with the largest positive biases for graph theory in the time period 1990-2010.

Journal	Bias
Journal of Graph Theory (0364-9024)	6.035
Discussiones Mathematicae. Graph Theory (1234-3099)	6.023
AKCE International Journal of Graphs and Combinatorics (0972-8600)	5.928
Journal of Combinatorial Theory. Series B (0095-8956)	5.774
Graphs and Combinatorics (0911-0119)	5.618
Applicable Analysis and Discrete Mathematics (1452-8630)	5.604
Ars Combinatoria. The Canadian Journal of Combinatorics (0381-7032)	5.297
The Australasian Journal of Combinatorics (1034-4942)	5.265
JCMCC. The Journal of Combinatorial Mathematics and Combinatorial Computing (1983-0823)	5.238
MATCH - Communications in Mathematical and in Computer Chemistry (0340-6253)	5.233

Table 10 Journals with the largest negative biases for graph theory in the time period 1990-2010.

Journal	Bias
International Journal of Solids and Structures (0020-7683)	-7.765
Journal of Differential Equations (0022-0396)	-7.427
International Journal of Modern Physics A. Particles and Fields, Gravitation and Cosmology (0217-751X)	-7.127
Classical and Quantum Gravity. An International Journal of Gravitational Physics, Cosmology, Geometry and Field Theory (0264-9381)	-6.982
Modern Physics Letters A. Particles and Fields, Gravitation, Cosmology, Nuclear Physics (0217-7323)	-6.797
Systems & Control Letters (0167-6911)	-6.516
Applicable Analysis. An International Journal (0003-6811)	-6.472
Acta Arithmetica (0065-1036)	-6.357
Nonlinear Analysis. Theory, Methods & Applications. Series A: Theory and Methods. An International Multidisciplinary Journal (0362-546X)	-6.124
Annals of Physics (0003-4916)	-6.070

For further analysis we used the network $\mathbf{WM}^{[05C]}$, which is a network \mathbf{WM} with the second set of nodes restricted so that only MSCs from graph theory remain. In order to get the network $\mathbf{WM}^{[05C]}$, we first made a partition of classifications σ in which all 05C classifications are in one class and the other classifications are in another class. With the partition σ we extracted the subnetwork \mathbf{WM}_σ from the network \mathbf{WM} . The network \mathbf{WM}_σ contains all works and only 05C classifications. Then we determined the outdegree partition of works τ and removed from the network \mathbf{WM}_σ all nodes (works) with outdegree 0. The works with outdegree greater than 0 have at least one MSC from graph theory. The resulting network is $\mathbf{WM}^{[05C]}$.

We used the partition of works τ on networks \mathbf{WA} , \mathbf{WJ} , and \mathbf{WK} to extract networks $\mathbf{WA}^{[05C]}$, $\mathbf{WJ}^{[05C]}$, and $\mathbf{WK}^{[05C]}$, respectively, in which are included only works about graph theory and their authors, journals in which they were published, and used keywords.

We examined degrees of nodes in the obtained networks to determine distributions of different data as we did for the whole set of works in Section 3.

The distribution of works about graph theory by the number of authors is presented in Fig. 2 in Section 3 in the top diagram. The distribution is shown in the same diagram as the distribution of all analyzed works by the number of authors. Both distributions are similar. More than $\frac{1}{3}$ of all works (34.34%) were written by a single author and even more (36.86%) by a pair of authors. 19 works do not have any author determined.

The distribution of works about graph theory by the number of keywords is shown in Fig. 2 in the middle. This distribution has a higher peak at a lower value (7) than the distribution of all analyzed works by the number of assigned keywords. Approximately 65% of all works have the number of keywords between 5 and 10.

The distribution of works about graph theory by the number of MSCs is shown in Fig. 2 at the bottom figure. This distribution is also almost the same as the distribution of all analyzed works by the number of MSCs. Approximately one third of all works (30.27%) were classified with two MSCs and 47.38% of all works were classified with one or three MSCs.

The distribution of authors by the number of works about graph theory they co-authored is displayed in Fig. 3 in Section 3 in the top figure together with the distribution of authors by the number of all works they co-authored. For example, the

lighter dot in the upper left corner represents 13801 authors that in the time-period 1990-2010 each co-authored only one work. Both distributions looks alike.

The distribution of keywords by the number of works about graph theory they describe is shown in Fig. 3 in the second figure together with the distribution of keywords by the number of all works they describe. The lighter dot in the upper left corner represents 6231 keywords that each was used in description of only one work in the time-period of 1990-2010. Again, the shape of the distribution is typical for the power law $f_n = cn^{-\alpha}$ for $\alpha = 1.72$, which is a bit smaller than the value of α for the distribution of keywords according to all works ($\alpha = 1.85$).

And finally, the distribution of MSCs by the number of works about graph theory that were classified with them is shown in Fig. 3 at the bottom figure together with the distribution of MSCs by the number of all analyzed works that were classified with given MSCs. The lighter dot in the upper left corner represents 1336 MSCs, that each was included in the classification of only one work about graph theory in the time-period of 1990-2010. This distribution has a higher value at the beginning (at value 1) and drops faster than the distribution of MSCs according to all analyzed works.

The sequence of journals sorted in a decreasing order by the number of indexed works about graph theory in the time-period 1990-2010 is shown in Fig. 5 together with the sequence of journals in a decreasing order by the number of all indexed works in the time period 1990-2010. Journals on the left have published the largest numbers of works. The Bradford's graph form of the sequence of journals about graph theory coincide with the Bradford's graph form of the sequence of journals about all mathematics only in the end – on the right side.

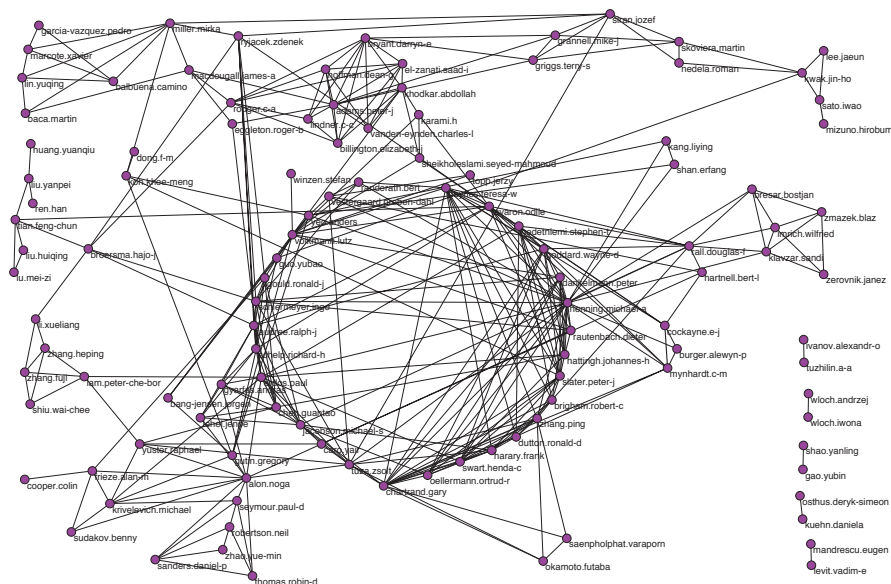
We used the partition of works τ on the network \mathbf{WA} to extract the network $\mathbf{WA}^{[05C]}$ in which are included only the works about graph theory and their authors. With the input degree partition of the second set of nodes in the network $\mathbf{WA}^{[05C]}$ we got the list of authors that published largest amounts of works about graph theory. Another way to see which authors published largest amounts of works about graph theory is to look at the values on the loops in the normalized collaboration network (Table 11). We checked the uniqueness of names in this list with the AMS Authors Search and only two names possibly represent more than one author: Liu, Guizhen (two authors) and Zhang, Ping (58 authors).

As we searched for the strongest collaboration ties in the collaboration network among all mathematicians, we did the same for graph theorists. We determined the normalized collaboration network $\mathbf{Ct}^{[05C]}$ for graph theorists – using the normalized $\mathbf{WA}^{[05C]}$ network. The p_S -core at level $t = 3.5$ is presented in Fig. 7. There are only few pairs of collaborators and one big group. One can notice stronger collaborations (darker and thicker links) inside subgroups of this group and these subgroups are linked to each other with weaker collaborations (lighter links).

Another way of identifying strong collaboration groups among graph theorists is using link islands. A link island in a network $\mathcal{N} = (\mathcal{V}, \mathcal{E}, w)$ is a subnetwork $\mathcal{M} = (\mathcal{U}, \mathcal{F}, w)$ such that there exists a spanning tree \mathcal{T} , such that the values of links with exactly one end node in \mathcal{U} are smaller or equal to the smallest value of links of the tree \mathcal{T} . The link islands determine the locally important subnetworks. In Fig. 8, 9, 10 three link islands of the size between 10 and 30 for the graph the-

Table 11 The list of 20 authors with the largest contributions to their works about graph theory in the time-period 1990-2010.

i	Author	cn_{ij}	Total	K_i
1	Volkman, Lutz	123.55	216	0.428
2	Henning, Michael A.	110.87	232	0.522
3	Liu, Yanpei	102.42	196	0.478
4	Alon, Noga	85.39	177	0.518
5	Tuza, Zsolt	77.05	150	0.486
6	Zhu, Xuding	76.85	132	0.418
7	Gutman, Ivan	70.13	143	0.510
8	Thomassen, Carsten	68.83	82	0.161
9	Mohar, Bojan	67.85	111	0.389
10	Liu, Guizhen	67.28	137	0.509
11	Liu, Bolian	63.67	119	0.465
12	Klavžar, Sandi	62.74	129	0.514
13	Bollobás, Béla	62.53	143	0.563
14	Zhang, Ping	60.47	157	0.615
15	Li, Xueliang	60.40	136	0.556
16	Rödl, Vojtěch	57.98	146	0.603
17	Zhang, Zhongfu	57.07	162	0.648
18	McKee, Terry A.	54.98	64	0.141
19	Zelinka, Bohdan	54.50	57	0.044
20	Yuster, Raphael	52.56	79	0.335

**Fig. 7** The p_5 -core at level $t = 3.50$ in the collaboration network C_t of graph theorists.

orists in the normalized collaboration network C_t are presented. For details see the slides Zaveršnik, M., & Batagelj, V. (2004): *Islands* that were presented on the XXIV. International Sunbelt Social Network Conference in Portorož, Slovenia, available at <http://vlado.fmf.uni-lj.si/pub/networks/doc/sunbelt/islands.pdf>.

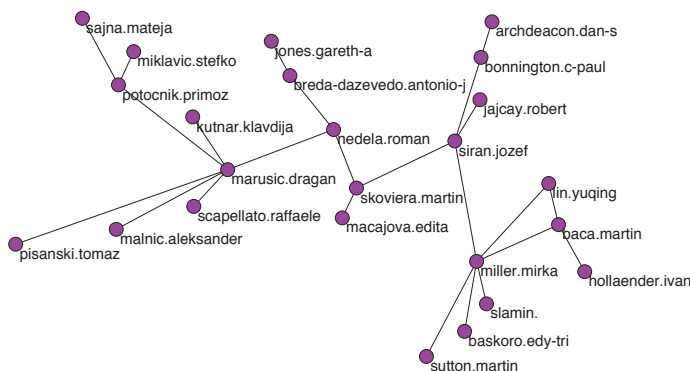


Fig. 8 A link island of graph theorists in the normalized collaboration network **Ct** with a subgroup of Slovenian and Slovak graph theorists.

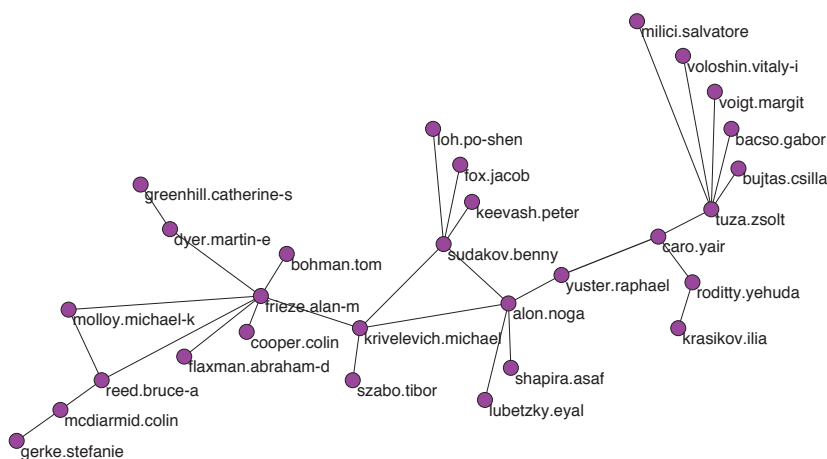


Fig. 9 A link island of graph theorists in the normalized collaboration network **Ct** with Noga Alon in the middle.

Many works about graph theory have also some other classifications besides graph theory. Multiple classifications for a work are representing the interdisciplinarity of a work. We used the network $\mathbf{WM}^{[05C]}$ to get the list of classification that coappeared within works about graph theory the most. To get this information, we used a partition of works τ in the network \mathbf{WM} to get a subnetwork of works about graph theory and all classifications $\mathbf{W}_\tau\mathbf{M}$. We shrunk the set of works into a single node. Classifications with the largest weighted input degrees represent mathematical areas that work interdisciplinary with the graph theory the most. These classifications are listed in Table 12. Each classification is defined with the MSC code in the first column, its name in the third column and a 2-char classification name (mathematical discipline). Classifications are arranged according to the value in the last column

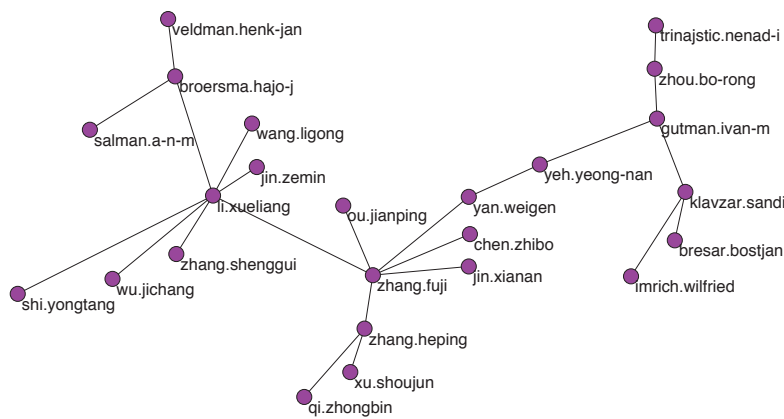


Fig. 10 A link island of graph theorists in the normalized collaboration network C_t with mostly Asian names.

– the number of works about the graph theory that were classified also with those classifications.

Table 12 A list of MSCs which coappeared most frequently with the graph theory classifications in the time-period 1990-2010.

MSC code	2-char MSC name	MSC name	No. of works
68R10	Computer science	Graph Theory	3528
68Q25	Computer science	Analysis of algorithms and problem complexity	1140
90C35	Operations research, mathematical programming	Programming involving graphs or networks	868
92E10	Biology and other natural sciences	Molecular structure	591
90C27	Operations research, mathematical programming	Combinatorial optimization	572
60C05	Probability theory and stochastic processes	Combinatorial probability	521
05A15	Combinatorics	Exact enumeration problems, generating functions	492
15A18	Linear and multilinear algebra; matrix theory	Eigenvalues, singular values, and eigenvectors	392
57M15	Manifolds and cell complexes	Relations with graph theory	381
05B35	Combinatorics	Matroids, geometric lattices	334
94C15	Information and communication, circuits	Applications of graph theory	317
68W25	Computer science	Approximation algorithms	315
05E30	Combinatorics	Association schemes, strongly regular graphs	312
06A07	Order, lattices, ordered algebraic structures	Combinatorics of partially ordered sets	291
90B10	Operations research, mathematical programming	Network models, deterministic	283
20B25	Group theory and generalizations	Finite automorphism groups of algebraic, geometric, or combinatorial structures	280
20D60	Group theory and generalizations	Arithmetic and combinatorial problems	275
68M10	Computer science	Network design and communication	274
91A43	Game theory, economics, social and behavioral sciences	Games involving graphs	237
05B20	Combinatorics	Matrices	222

Records for most of the works contain information about keywords. Some keywords are common in all areas of mathematics and some are used only in few areas. With a right weightening of keywords we can sort them by their importance for different areas of mathematics. We used the TF-IDF weightening (Robertson, 2004). Areas of mathematics can be determined by MSCs. We multiplied networks **MW** and **WK** in order to obtain the network **MK**. MSCs were shrinked according to 2-char MSC codes.

All keywords, used in all areas get the value zero in TF-IDF weightening. Others get values:

$$\text{TF-IDF}(\text{keyword}, \text{MSC}) = \text{TF}(\text{keyword}, \text{MSC}) \times \text{IDF}(\text{keyword})$$

$$\text{TF}(\text{keyword}, \text{MSC}) = \frac{\text{Value on the link between keyword and MSC}}{\text{Sum of values of all links from MSC}}$$

$$\text{IDF}(\text{keyword}) = \log \frac{\text{No. of MSCs}}{\text{No. of MSCs linked to keyword}}$$

There are 12460 keywords with a non-zero TF-IDF value linked to the MSCs of graph theory. The largest values have the keywords listed in Table 13. In the table are also listed absolute frequencies of keywords within graph theory and within all mathematics.

Table 13 A list of keywords with the highest TF-IDF value for the 3-char MSC 05C in the time-period 1990-2010.

Keyword	No. of appearances within graph theory	No. of all appearances	TF-IDF value ($\cdot 10^{-4}$)
Coloring	4133	6676	45.24
Digraph	4049	5695	44.32
Chromatic	3958	5138	43.32
Subgraph	2298	3473	35.31
Domination	2500	3883	27.36
Clique	1788	3169	25.29
Vertex	4611	12981	25.23
Hypergraph	2258	3808	24.71
Bipartite	2776	5045	24.08
Tournament	1335	2341	21.92
Matching	1038	1891	17.04
Label	1960	4656	17.00
Ramsey	1305	3148	16.58
Claw	625	715	15.02
Colour	1278	2385	13.99
Girth	736	954	13.93
Connectivity	2528	5462	13.83
Hamiltonicity	465	551	11.96
Match	2159	15140	11.82
Chordal	738	1384	11.34

6 Conclusions

The bibliographic data can be analyzed in many ways. In this paper we present some network analysis approaches applied to the Zentralblatt MATH database that stores information about mathematical publications. Through the results of our analysis of the ZB data from a time period 1990-2010 we conclude that mathematicians tend to work alone or in small groups. They also work in a specific area of mathematics. This can be seen from the small number of MSCs that classified each work and the small number of keywords per work.

Because the data entries in the database are only partially standardized there are some problems with the data. These problems can cause irregularities in the results. We solved some of the problems (for example the unification of journals) and partially solved some other problems (for example the unification of the names of authors).

We took a closer look at works about graph theory and determined journals that are ‘friendly’ to graph theory, the best graph theorists according to their contribution to the works they co-authored, other areas of mathematics that are closely connected to the graph theory through publications, and the keywords characteristic for the graph theory.

The network multiplication of compatible two-mode networks allows us to compute different derived networks. A network $\mathbf{AJ} = \mathbf{AW} * \mathbf{WJ}$ stores the information of the number of indexed works that were written by some author and published in some journal. This network can be analyzed or used further to produce new networks. One possibility is to multiply it by its transpose and obtain the network $\mathbf{JJ} = b(\mathbf{JA} * \mathbf{AJ})$. Two journals in this network are linked if there exists an author that published at least one indexed work in both journals. Another possibility is to use binarized networks: $\mathbf{JJ}_A = b(\mathbf{JA}) * b(\mathbf{AJ})$. In it, the weight of a link between two journals is equal to the number of authors that published in both journals. Using approaches presented in this paper, we could analyze similarities among indexed journals.

This is just an example of what could be done in the network analysis of the ZB data in the future. In our analysis we did not consider the information about the publication year. We plan to do the temporal analysis of the ZB data and to present the results in another paper.

Acknowledgements We thank prof. Bernd Wegner and his associates at FIZ Karlsruhe for providing the data, and prof. Tomaž Pisanski and dr. Boris Horvat for their joint part of the work on this project. We also thank Selena Praprotnik and anonymous referees for checking the text and suggesting several improvements.

The first author was financed in part by the European Union, European Social Fund. The work was supported in part by the ARRS, Slovenia, grant J5-5537, as well as by a grant within the EUROCORES Programme EUROGIGA (project GReGAS) of the European Science Foundation.

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